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## Vibration control of vehicle seat integrating with chassis suspension and driver body model

Haiping Du

*University of Wollongong*, [hdu@uow.edu.au](mailto:hdu@uow.edu.au)

Weihua Li

*University of Wollongong*, [weihuali@uow.edu.au](mailto:weihuali@uow.edu.au)

Nong Zhang

*University Of Technology Sydney*

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## Abstract

Vehicle seat suspension is one of very important components to provide ride comfort, in particular, commercial vehicles, to reduce driver fatigue due to long hours driving. This paper presents a study on active control of seat suspension to reduce vertical vibration transmitted from uneven road profile to driver body. The control problem will be firstly studied by proposing an integrated seat suspension model which includes vehicle chassis suspension, seat suspension, and driver body model. This is a new concept in the field of study because most of the current active and semi-active seat suspension studies only consider seat suspension or seat suspension with human body model, and road disturbance is generally assumed to be applied to the cabin floor directly. Controller design based an integrated model will enable the seat suspension to perform in a scenario where vibration caused by road disturbance is transmitted from wheel to seat frame and ride comfort performance is evaluated in terms of human body instead of seat frame acceleration. A static output feedback controller is then designed for the seat suspension with using measurement available signals. Driver mass variation and actuator saturation are also considered in the controller design process. The conditions for designing such a controller are derived in terms of linear matrix inequalities (LMIs). Finally, numerical simulations are used to validate the effectiveness of the proposed control strategy. It is shown from the driver body acceleration responses under both bump and random road disturbances that the newly designed seat suspension can improve vehicle ride comfort regardless of driver body mass variation.

## Keywords

suspension, driver, body, model, control, vibration, seat, vehicle, integrating, chassis

## Disciplines

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# Vibration Control of Vehicle Seat Integrating with Chassis Suspension and Driver Body Model

Haiping Du<sup>1,\*</sup>, Weihua Li<sup>2</sup> and Nong Zhang<sup>3</sup>

<sup>1</sup>School of Electrical, Computer and Telecommunications Engineering, University of Wollongong, Wollongong, NSW 2552, Australia

<sup>2</sup>School of Mechanical, Material, and Mechatronics Engineering, University of Wollongong, Wollongong, NSW 2552, Australia.

<sup>3</sup>Mechatronics and Intelligent Systems, Faculty of Engineering, University of Technology, Sydney, Broadway, NSW 2007, Australia.

**Abstract:** Vehicle seat suspension is one of very important components to provide ride comfort, in particular, commercial vehicles, to reduce driver fatigue due to long hours driving. This paper presents a study on active control of seat suspension to reduce vertical vibration transmitted from uneven road profile to driver body. The control problem will be firstly studied by proposing an integrated seat suspension model which includes vehicle chassis suspension, seat suspension, and driver body model. This is a new concept in the field of study because most of the current active and semi-active seat suspension studies only consider seat suspension or seat suspension with human body model, and road disturbance is generally assumed to be applied to the cabin floor directly. Controller design based an integrated model will enable the seat suspension to perform in a scenario where vibration caused by road disturbance is transmitted from wheel to seat frame and ride comfort performance is evaluated in terms of human body instead of seat frame acceleration. A static output feedback controller is then designed for the seat suspension with using measurement available signals. Driver mass variation and actuator saturation are also considered in the controller design process. The conditions for designing such a controller are derived in terms of linear matrix inequalities (LMIs). Finally, numerical simulations are used to validate the effectiveness of the proposed control strategy. It is shown from the driver body acceleration responses under both bump and random road disturbances that the newly designed seat suspension can improve vehicle ride comfort regardless of driver body mass variation.

**Key words:** Seat suspension control, integrated model, static output feedback, actuation saturation, load variation.

## 1. INTRODUCTION

Seat suspension has been commonly accepted in commercial vehicles for industrial, agricultural and other transport purposes (Choi *et al.* 2000) to provide driver ride comfort, to reduce driver fatigue due to long hour driving or exposure to severe working environment such as rough road condition, and to improve driver safety and health (Tiemessen *et al.* 2007). Study on optimisation and control of seat suspensions for reducing vertical vibration has been an active topic for decades. Three main types of seat suspensions, i.e., passive seat suspension, semi-active

seat suspension, and active seat suspension, have been presented so far. The study on passive seat suspension mainly focuses on parameter optimisation for the spring stiffness and the damping coefficient. With the development of magnetorheological (MR) or electrorheological (ER) dampers, semi-active control of seat suspension has been proposed to provide variable damping force with less power consumption (Choi *et al.* 2000; Choi and Han 2007). The study on active seat suspension mainly focuses on developing advanced control strategies or applying different types of actuators to improve seat suspension performance with

\*Corresponding author. Email address: hdu@uow.edu.au; Fax: 61-2-42213236; Tel: 61-2-42214610.

taking account of issues like actuator saturation, load variation, time delay, and reliability, etc. (Maciejewski *et al.* 2010; Zhao *et al.* 2010a; Zhao *et al.* 2010b; Sun *et al.* 2011). Among these three types of seat suspensions, active seat suspension is able to provide the best ride comfort performance, and therefore, receives much more attention in recent years.

In addition to seat suspension, vehicle chassis suspension has been extensively studied for decades (Hrovat 1997). Vehicle chassis suspension is, in fact, designed as a primary suspension for all the vehicles to provide ride comfort, road holding, and other dynamic functions. However, it is noticed that most of the current active/semi-active seat suspension and active/semi-active vehicle chassis suspension are designed/studied separately though their common function is to improve vehicle ride comfort performance.

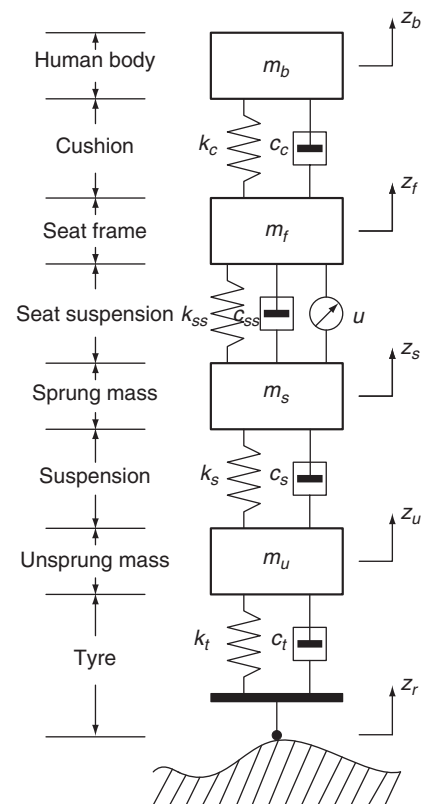
In this paper, active control strategy will be applied to reduce the vertical vibration of driver body when sitting in a running vehicle. The main contributions are given in the following. Firstly, the control problem will be studied by considering an integrated suspension model which includes vehicle chassis suspension, seat suspension, and human body model. As vehicle chassis suspension itself is designed to isolate vibration from road disturbance to vehicle body, considering vehicle chassis suspension functions when designing a seat suspension system will obtain more accurate information on vibration sources and therefore reduce control effort for seat suspension in terms of attenuated excitation input. In addition, human body model is necessary to be included as acceleration reduction should be evaluated on human body not on the cabin floor. Secondly, as a controller needs to use available information as feedback signals, state feedback may not be able to do this when some signals are not available for measurement, for example, head displacement, cushion displacement, etc. The shortcoming of dynamic output feedback control is that its order will be higher, in particular, when high degree-of-freedom (DOF) of human-body model will be considered, which results in either higher order controller, which is hard to be implemented, or no feasible solution. From this point of view, this paper will present static output feedback control design for seat suspension. Some possible configurations in terms of available measurements will be further studied. Simulation results show that some signals are not helpful in vibration reduction even they are measurement available. Thirdly, driver load variation will be considered as different drivers may have different weights. Taking weight variation into account will make the controller have similar performance for different drivers. At last, actuator

saturation is considered. This is a practical constraint that needs to be dealt with when implementing a controller in practice.

## 2. INTEGRATED VEHICLE SEAT SUSPENSION MODEL

The integrated vehicle seat suspension model includes a quarter-car suspension model, a seat suspension model, and a driver body model as shown in Figure 1, where  $m_s$  is the sprung mass, which represents the car chassis;  $m_u$  is the unsprung mass, which represents the wheel assembly;  $m_f$  is the seat frame mass; and  $m_b$  is the driver body mass.  $z_u$ ,  $z_s$ ,  $z_f$ ,  $z_c$  and  $z_b$  are the displacements of the corresponding masses, respectively;  $z_r$  is the road displacement input.  $c_s$  and  $k_s$  are damping and stiffness of the car suspension system, respectively;  $k_t$  and  $c_t$  stand for compressibility and damping of the pneumatic tyre, respectively;  $c_{ss}$ ,  $c_c$ ,  $k_{ss}$  and  $k_c$  are damping and stiffness of seat suspension and seat cushion, respectively.  $u$  represents the active control force applied to the seat suspension.

It is assumed that only the vertical motion of vehicle is concerned in the paper, and both pitching and rolling motions are neglected. Therefore the quarter-car suspension model, which has been used extensively in



**Figure 1.** Integrated car suspension, seat suspension, and driver body model

the literature, can sufficiently capture the important characteristics of more detailed models. The seated human body exposed to vibration is a sophisticated dynamic system whose mechanical properties are complex. Considering the trade-off between the complexity and the accuracy for the study of biodynamic responses of seated human subjected to vertical whole body vibration as well as getting a better insight of the controller design, the driver body model is assumed to be a rigid dummy mass which is rigidly contacted with the seat. Without loss of generality, to derive the equation of motion of the model, the effect of the actuator dynamics is neglected and the actuator is assumed to be an ideal force generator.

The dynamic vertical motion of equations for the quarter-car suspension, seat suspension, and driver body are given by

$$\begin{aligned}
 m_u \ddot{z}_u &= -k_t(z_u - z_r) - c_t(\dot{z}_u - \dot{z}_r) \\
 &\quad + k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) \\
 m_s \ddot{z}_s &= -k_s(z_s - z_u) - c_s(\dot{z}_s - \dot{z}_u) \\
 &\quad + k_{ss}(z_f - z_s) + c_{ss}(\dot{z}_f - \dot{z}_s) + u \\
 m_f \ddot{z}_f &= -k_{ss}(z_f - z_s) - c_{ss}(\dot{z}_f - \dot{z}_s) \\
 &\quad + k_c(z_c - z_f) + c_c(\dot{z}_c - \dot{z}_f) - u \\
 m_b \ddot{z}_b &= -k_c(z_c - z_f) - c_c(\dot{z}_c - \dot{z}_f)
 \end{aligned} \quad (1)$$

By defining the following set of state variables  $x_1 = z_u, x_2 = \dot{z}_u, x_3 = z_s - z_u, x_4 = \dot{z}_s - \dot{z}_u, x_5 = z_f - z_s, x_6 = \dot{z}_f - \dot{z}_s, x_7 = z_c - z_f, x_8 = \dot{z}_c - \dot{z}_f$  and state vector  $x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8]^T$ , we can write the dynamic Eqn 1 into a state-space form as

$$\dot{x} = Ax + B_w w + B\bar{u} \quad (2)$$

where

$$A = \begin{bmatrix}
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 -\frac{k_t}{m_u} & -\frac{c_t + c_s}{m_u} & \frac{k_s}{m_u} & \frac{c_s}{m_u} & 0 & 0 & 0 & 0 \\
 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & \frac{c_s}{m_s} & -\frac{k_s}{m_s} & -\frac{c_s + c_{ss}}{m_s} & \frac{k_{ss}}{m_s} & \frac{c_{ss}}{m_s} & 0 & 0 \\
 0 & 0 & 0 & -1 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & \frac{c_{ss}}{m_f} & -\frac{k_{ss}}{m_f} & -\frac{c_{ss} + c_c}{m_f} & \frac{k_c}{m_f} & \frac{c_c}{m_f} \\
 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \\
 0 & 0 & 0 & 0 & 0 & \frac{c_c}{m_b} & -\frac{k_c}{m_b} & -\frac{c_c}{m_b}
 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{m_s} & 0 & -\frac{1}{m_f} & 0 & 0 \end{bmatrix}^T, \\
 B_w = \begin{bmatrix} -1 & \frac{C_t}{m_u} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$

$u$  is the active control input of the seat suspension system and will be defined in the next section, and  $w = \dot{z}_r$  is the road disturbance.

In practice, all the actuators are limited by their physical capabilities, and hence, actuator saturation needs to be considered for active control of seat suspension (Zhao *et al.* 2010b). Taking actuator saturation into account, Eqn 2 is modified as

$$\dot{x} = Ax + B_w w + B\bar{u} \quad (3)$$

where  $\bar{u} = \text{sat}(u)$ , and  $\text{sat}(u)$  is a saturation function of control input  $u$  defined as

$$\text{sat}(u) = \begin{cases} -u_{\lim} & \text{if } u < -u_{\lim} \\ -u_{\lim} & \text{if } -u_{\lim} \leq u \leq u_{\lim} \\ u_{\lim} & \text{if } u > u_{\lim} \end{cases} \quad (4)$$

As the varying driver body mass  $m_b$  is actually bounded by its minimum value  $m_{b\min}$  and its maximum value  $m_{b\max}$  in a real operation, it is not difficult to represent the uncertain vehicle mass by

$$1/m_b = M_1(\xi)m_{s\max} + M_2(\xi)m_{s\min}$$

where  $\xi = 1/m_b, m_{s\max} = 1/m_{b\min}, m_{s\min} = 1/m_{b\max}$ ,  $M_1(\xi)$  and  $M_2(\xi)$  are defined as

$$M_1(\xi) = \frac{1/m_b - m_{s\min}}{m_{s\max} - m_{s\min}}, M_2(\xi) = \frac{m_{s\max} - 1/m_b}{m_{s\max} - m_{s\min}}$$

Hence, the parameter varying system for the integrated model can be expressed as

$$\dot{x} = \sum_{i=1}^2 M_i(\xi) A_i x + B_w w + B\bar{u} = A_m x + B_w w + B\bar{u} \quad (5)$$

where  $A_m = \sum_{i=1}^2 M_i(\xi) A_i$ .

### 3. STATIC OUTPUT FEEDBACK CONTROLLER DESIGN

For seat suspension design, the performance on ride comfort is mainly described by the driver body acceleration (Zhao *et al.* 2010; Sun *et al.* 2011), and therefore, the driver body acceleration,

$$z = \ddot{z}_b = C_1 x \quad (6)$$

where  $C_1$  is composed of the last row of  $A$  in Eqn 2, is defined as the control output.

To achieve good ride comfort and make the controller performing adequately for a wide range of road disturbances, the  $L_2$  gain between the road disturbance input  $w$  and the control output  $z$ , which is defined as

$$\|T_{zw}\|_\infty = \sup_{\|w\|_2 \neq 0} \frac{\|z\|_2}{\|w\|_2} \quad (7)$$

where  $\|z\|_2^2 = \int_0^\infty z^T z dt$  and  $\|w\|_2^2 = \int_0^\infty w^T w dt$ , is chosen

as the performance measure. A small value of  $\|T_{zw}\|_\infty$  generally means a small value of driver body acceleration under the energy bounded road disturbances. The control objective is to design a controller

$$u = \sum_{i=1}^2 M_i(\xi) K_i C x = K_m C x \quad (8)$$

where  $K_m = \sum_{i=1}^2 M_i(\xi) K_i$ ,  $K_i$  is a constant feedback gain matrix to be designed such that the closed-loop system, which is composed by substituting Eqn 8 into Eqn 5, is asymptotically stable, and the performance measure Eqn 7 is minimised. Matrix  $C$  in Eqn 8 is used to define the available state variables. For example, if only the first state variable  $x_1$  is available for feedback, then  $C$  is defined as  $C = [1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]$ .

For designing the controller (Eqn 8), we define a Lyapunov function for the system (Eqn 5) as

$$V(x) = x^T P x, \quad (9)$$

where  $P$  is a positive definite matrix. By differentiating Eqn 9, we obtain

$$\begin{aligned} \dot{V}(x) &= \dot{x}^T P x + x^T P \dot{x} \\ &= \begin{bmatrix} A_m x + B \frac{1+\varepsilon}{2} u + B \left( \bar{u} - \frac{1+\varepsilon}{2} u \right) \\ + B_w w \end{bmatrix}^T P x \\ &\quad + x^T P \begin{bmatrix} A_m x + B \frac{1+\varepsilon}{2} u + B \left( \bar{u} - \frac{1+\varepsilon}{2} u \right) \\ + B_w w \end{bmatrix} \end{aligned} \quad (10)$$

By using the following inequalities

$$\begin{aligned} \left[ \bar{u} - \frac{1+\varepsilon}{2} u \right]^T \left[ \bar{u} - \frac{1+\varepsilon}{2} u \right] &\leq \left( \frac{1-\varepsilon}{2} \right)^2 u^T u, \\ \text{for } |u| &\leq \frac{u_{\lim}}{\varepsilon} \text{ and } \varepsilon > 0 \end{aligned} \quad (11)$$

$$X^T Y + Y^T X \leq \sigma X^T X + \sigma^{-1} Y^T Y \quad (12)$$

and the definition Eqn 8, we have

$$\begin{aligned} \dot{V}(x) &= x^T \left[ A_m^T P + P A_m + \left( B \frac{1+\varepsilon}{2} K_m C \right)^T \right. \\ &\quad \left. P + P B \frac{1+\varepsilon}{2} K_m C \right] x \\ &\quad + w^T B_w^T P x + x^T P B_w w + \sigma \left( \bar{u} - \frac{1+\varepsilon}{2} u \right)^T \\ &\quad \left( \bar{u} - \frac{1+\varepsilon}{2} u \right) + \sigma^{-1} x^T P B B^T P x \\ &\leq x^T \Theta x + w^T B_w^T P x + x^T P B_w w \end{aligned} \quad (13)$$

where

$$\begin{aligned} \Theta &= A_m^T P + P A_m + \left( B \frac{1+\varepsilon}{2} K_m C \right)^T P + P B \frac{1+\varepsilon}{2} K_m C \\ &\quad + \sigma \left( \frac{1+\varepsilon}{2} \right)^2 C^T K_m^T K_m C + \sigma^{-1} P B B^T P \end{aligned}$$

Adding  $z^T z - \gamma^2 w^T w$   $\gamma > 0$  to both sides of Eqn 13 yields

$$\begin{aligned} \dot{V}(x) + z^T z - \gamma^2 w^T w &= \begin{bmatrix} x^T & w^T \end{bmatrix} \\ &\quad \begin{bmatrix} \Theta + C_1^T C_1 & P B_1 \\ B_w^T P & -\gamma^2 I \end{bmatrix} \begin{bmatrix} x \\ w \end{bmatrix} \end{aligned} \quad (14)$$

Let us consider

$$\Pi = \begin{bmatrix} \Theta + C_1^T C_1 & P B_w \\ B_w^T P & -\gamma^2 I \end{bmatrix} < 0 \quad (15)$$

then  $\dot{V}(x) + z^T z - \gamma^2 w^T w \leq 0$ , and the  $L_2$  gain defined in Eqn 7 is less than  $\gamma > 0$  with the initial condition  $x(0) = 0$ . When the disturbance is zero, i.e.,  $w = 0$ , it can be inferred from Eqn 14 that if  $\Pi < 0$ , then  $\dot{V}(x) < 0$ , and the closed-loop system is quadratically stable.

Pre- and post-multiplying Eqn 15 by  $\text{diag}(P^{-1}, I)$  and its transpose, respectively, and defining  $Q = P^{-1}$ ,  $W C = C Q$ ,  $Y_m = K_m Q$ , the condition  $\Pi < 0$ , is equivalent to

$$\Sigma = \begin{bmatrix} QA_m^T + A_m Q + \frac{1+\varepsilon}{2} C^T Y_m^T B^T \\ + \frac{1+\varepsilon}{2} B Y_m C \\ + \sigma \left( \frac{1-\varepsilon}{2} \right)^2 C^T Y_m^T Y_m C + \sigma^{-1} B B^T \\ + Q C_1^T C_1 Q \\ * \\ -\gamma^2 I \end{bmatrix} B_w < 0 \quad (16)$$

Using the Schur complement,  $\Sigma < 0$  is equivalent to

$$\Lambda = \begin{bmatrix} QA_m^T + A_m Q + \frac{1+\varepsilon}{2} \\ [C^T Y_m^T B^T + B Y_m C] & C^T Y_m^T & Q C_1^T & B_w \\ + \sigma^{-1} B B^T & * & * & * \\ * & -\sigma^{-1} \left( \frac{1-\varepsilon}{2} \right)^2 I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (17)$$

By the definition  $A_m = \sum_{i=1}^2 M_i(\xi) A_i$ , and the fact that

$M_i(\xi) \geq 0$  and  $\sum_{i=1}^2 M_i(\xi) = 1$ ,  $\Lambda < 0$  is equivalent to

$$\begin{bmatrix} QA_i^T + A_i Q + \frac{1+\varepsilon}{2} \\ [C^T Y_i^T B^T + B Y_i C] & C^T Y_i^T & Q C_1^T & B_w \\ + \sigma^{-1} B B^T & * & * & * \\ * & -\sigma^{-1} \left( \frac{1-\varepsilon}{2} \right)^2 I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad i = 1, 2 \quad (18)$$

On the other hand, from Eqn 8, the constraint  $|u| \leq \frac{u_{\lim}}{\varepsilon}$  can be expressed as

$$\left| \sum_{i=1}^2 M_i(\xi) K_i C x \right| \leq \frac{u_{\lim}}{\varepsilon} \quad (19)$$

It is induced that if  $|K_i C x| \leq \frac{u_{\lim}}{\varepsilon}$ , then Eqn 19 holds.

Let us define a set of  $\Omega(K) = \left\{ x \mid x^T C^T K_i^T K_i C x \leq \left( \frac{u_{\lim}}{\varepsilon} \right)^2 \right\}$ ,

then the equivalent condition for an ellipsoid  $\Omega(P, \rho) = \{x \mid x^T P x \leq \rho\}$  being a subset of  $\Omega(K)$ , i.e.,  $\Omega(P, \rho) \subset \Omega(K)$ , is

$$K_i C \left( \frac{P}{\rho} \right)^{-1} C^T K_i^T \leq \left( \frac{u_{\lim}}{\varepsilon} \right)^2 \quad (20)$$

By the Schur complement, inequality Eqn 20 can be written as

$$\begin{bmatrix} \left( \frac{u_{\lim}}{\varepsilon} \right)^2 I & K_i C \left( \frac{P}{\rho} \right)^{-1} \\ * & \left( \frac{P}{\rho} \right)^{-1} I \end{bmatrix} \geq 0 \quad (21)$$

Using definitions of  $Q = P^{-1}$ ,  $WC = CQ$ , and  $Y_m = K_m Q$ , inequality Eqn 21 is equivalent to

$$\begin{bmatrix} \left( \frac{u_{\lim}}{\varepsilon} \right)^2 I & Y_i C \\ * & \rho^{-1} Q \end{bmatrix} \geq 0 \quad (22)$$

Now, we consider the computational algorithm for implementing the proposed controller. It is observed from above that the controller design is the feasibility problem of linear matrix inequalities (LMIs) Eqns 18 and 22 with equality constraint  $WC = CQ$ . We now convert the equality constraint problem to the LMI problem (Du and Zhang 2009). Consider the equality constraint  $WC = CQ$ , it can be equivalently converted to

$$\text{tr}[(WC - CQ)^T (WC - CQ)] = 0 \quad (23)$$

By introducing the condition

$$(WC - CQ)^T (WC - CQ) \leq \mu I \quad (24)$$

which is equivalent to

$$\begin{bmatrix} -\mu I & (WC - CQ)^T \\ * & -I \end{bmatrix} \leq 0 \quad (25)$$

by means of the Schur complement, the design of the static output feedback controller can be changed to a



problem of finding a global solution to the minimisation problem of:

$$\begin{aligned} &\text{minimise } \mu \text{ subject to LMIs Eqns 18,} \\ &22, \text{ and 25 for } i = 1, 2 \end{aligned} \quad (26)$$

This minimisation problem can be solved by using the LMI Toolbox in Matlab. If  $\mu$  equals to zero, then the solutions will satisfy the LMIs Eqns 18 and 22, and the equality  $WC = CQ$ , and then, the static output feedback controller can be obtained as  $K_i = Y_i W^{-1}$ .

**Remark 1:** For active control of vehicle seat suspensions, unavoidable time delays may appear when actuators, such as hydraulic actuators, are applied to build up the required control forces. The time delays may cause poor system performance or even possible instability of the closed-loop system. To overcome the actuator time-delay problem, the time delays can be considered into the controller design process by using different methods, see, for example, references (Zhao *et al.* 2010b; Du and Zhang 2007; Du *et al.* 2008; Gao *et al.* 2010). As some electro-magnetic actuators which are potentially used for active control of vehicle seat suspensions, such as permanent magnet synchronous motor (PMSM) (Guclu and Gulez 2008), have fast dynamic response characteristics such that time delay effects can be ignored, and the main focus of this paper is to present the integrated seat suspension model, the actuator time-delay issue is not further discussed in this paper.

#### 4. NUMERICAL SIMULATIONS

Numerical simulations are conducted in this section to show the effectiveness of the proposed integrated seat suspension model and control strategy for improving driver ride comfort. The parameters used in the simulations are listed in Table 1, where the quarter-car suspension parameters have been optimised in terms of driver body acceleration in (Kuznetsov *et al.* 2011) and the seat suspension and driver body model parameters are referred to (Choi and Han 2007).

In the simulation, the actuator force limitation for the seat suspension is considered as  $u_{\text{lim}} = 1000$  N. Driver body mass is assumed to be in the range of

40 kg to 100 kg, and the nominal mass is 70 kg as listed in Table 1.

To validate the suspension performance in time-domain, a typical road disturbance, i.e., bump road disturbance, will be considered in the simulation and applied to the vehicle wheel. The ground displacement for an isolated bump in an otherwise smooth road surface is given by

$$z_r = \begin{cases} \frac{a}{2} \left( 1 - \cos \left( \frac{2\pi v_0}{l} t \right) \right), & 0 \leq t \leq \frac{l}{v_0} \\ 0, & t > \frac{l}{v_0} \end{cases}$$

where  $a$  and  $l$  are the height and the length of the bump,  $v_0$  is vehicle forward speed. We choose  $a = 0.1$  m,  $l = 2$  m, and  $v_0 = 30$  km/h in the simulation.

To show the effectiveness and advance of the proposed control strategy, several different controllers will be designed and compared. At first, we design a state feedback controller with choosing the scalars  $\varepsilon = 0.9$  and  $\rho = 10^{-3}$  as

$$K = 10^5 \times [0.2221 \ 0.0005 \ 0.2341 \ 0.0191 \ 0.0910 \ 0.0484 \ -2.4017 \ 0.3043].$$

The state feedback controller is designed by setting  $C$  as an identity matrix, and solving LMIs Eqns 18 and 22 for  $i = 1$ . This controller does not consider the driver body mass variation. Then, we design different static output feedback controllers with assuming different measurable state variables. When assuming both seat suspension displacement and velocity are measurable, we define output matrix as  $C = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0; 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$ , and the static output feedback controller is obtained with choosing the scalars  $\varepsilon = 0.993$  and  $\rho = 1$  as

$$K_1 = 10^4 \times [0 \ 0 \ 0 \ 0 \ -0.5008 \ 1.7497 \ 0 \ 0] \text{ and } K_2 = 10^4 \times [0 \ 0 \ 0 \ 0 \ -0.4326 \ 2.2141 \ 0 \ 0].$$

The bump responses of the driver body acceleration for the integrated seat suspension system with the above-designed two controllers are compared in Figure 2, where Passive means no controller has been used. It can be seen from Figure 2 that the state feedback control achieves the best performance on ride comfort in terms of the peak value of driver body acceleration, and the static output feedback controller achieves similar performance to the state feedback controller in spite of its simple structure.

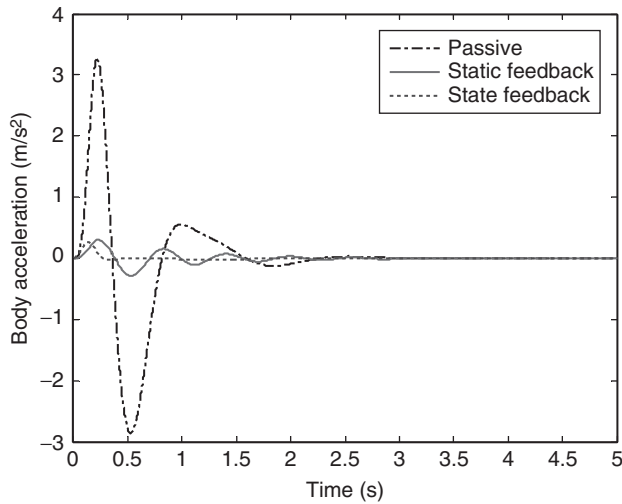
We now consider some other possible cases for designing a static output feedback controller.

**Case 1:** Car suspension displacement and velocity are measurable, output matrix is defined as  $C = [0 \ 0 \ 1 \ 0 \ 0 \ 0]$

**Table 1. Parameter values of the proposed suspension model**

Mass	kg	Damping	Ns/m	Spring	N/m
$m_u$	20	$c_t$	0	$k_t$	180000
$m_s$	300	$c_s$	2000	$k_s$	10000
$m_f$	20	$c_{ss}$	1080	$k_{ss}$	7414.86
$m_b$	70	$c_c$	152.8	$k_c$	8228.78





**Figure 2.** Bump responses on driver body acceleration for different control systems

$0 \ 0; \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0]$ , and the static output feedback controller is obtained as

$$K_1 = 10^3 \times [0 \ 0 \ -2.2272 \ 0.6235 \ 0 \ 0 \ 0 \ 0]; K_2 = 10^3 \times [0 \ 0 \ -0.2027 \ 0.6273 \ 0 \ 0 \ 0 \ 0].$$

**Case 2:** Car suspension displacement and seat suspension displacement are measurable, output matrix is defined as  $C = [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]$ , and the static output feedback controller is obtained as

$$K_1 = 10^3 \times [0 \ 0 \ 0.6746 \ 0 \ -4.7360 \ 0 \ 0 \ 0]; K_2 = 10^3 \times [0 \ 0 \ 4.6527 \ 0 \ -2.8969 \ 0 \ 0 \ 0].$$

**Case 3:** Car suspension velocity and seat suspension velocity are measurable, output matrix is defined as  $C = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$ , and the static output feedback controller is obtained as

$$K_1 = 10^5 \times [0 \ 0 \ 0 \ -0.1265 \ 0 \ 4.2378 \ 0 \ 0]; K_2 = 10^5 \times [0 \ 0 \ 0 \ -0.1066 \ 0 \ 3.5590 \ 0 \ 0].$$

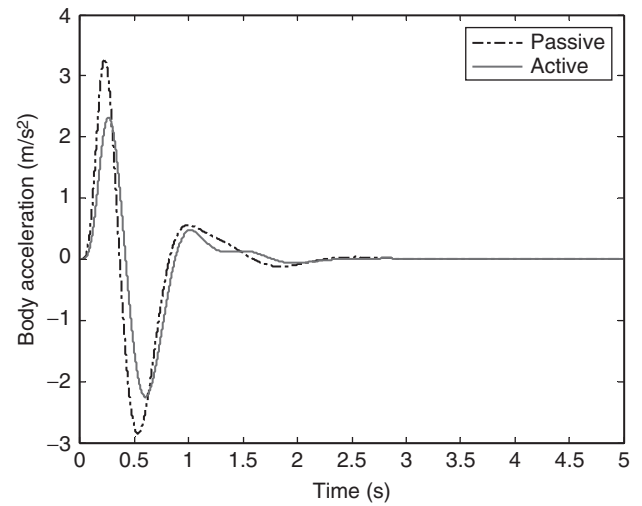
**Case 4:** Car suspension velocity and seat suspension displacement are measurable, output matrix is defined as  $C = [0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0]$ , and the static output feedback controller is obtained as

$$K_1 = 10^3 \times [0 \ 0 \ 0 \ 1.0029 \ -6.2319 \ 0 \ 0 \ 0]; K_2 = 10^3 \times [0 \ 0 \ 0 \ 0.6812 \ -5.5437 \ 0 \ 0 \ 0].$$

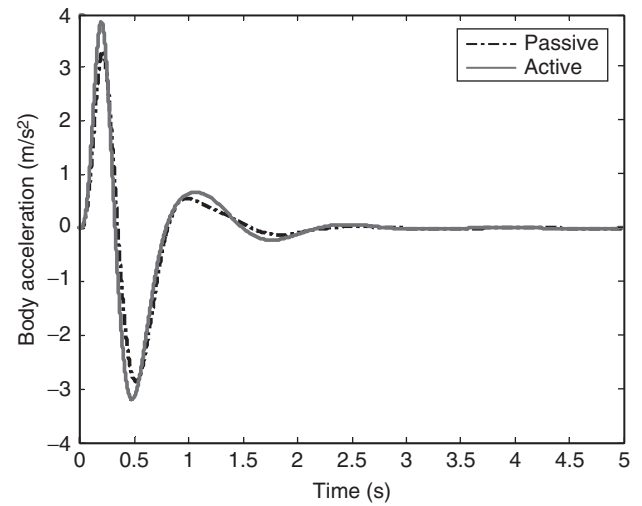
**Case 5:** Car suspension displacement and seat suspension velocity are measurable, output matrix is defined as  $C = [0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0; \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]$ , and the static output feedback controller is obtained as

$$K_1 = 10^3 \times [0 \ 0 \ 5.5181 \ 0 \ 0 \ 3.4603 \ 0 \ 0]; K_2 = 10^3 \times [0 \ 0 \ 5.0012 \ 0 \ 0 \ 3.0509 \ 0 \ 0].$$

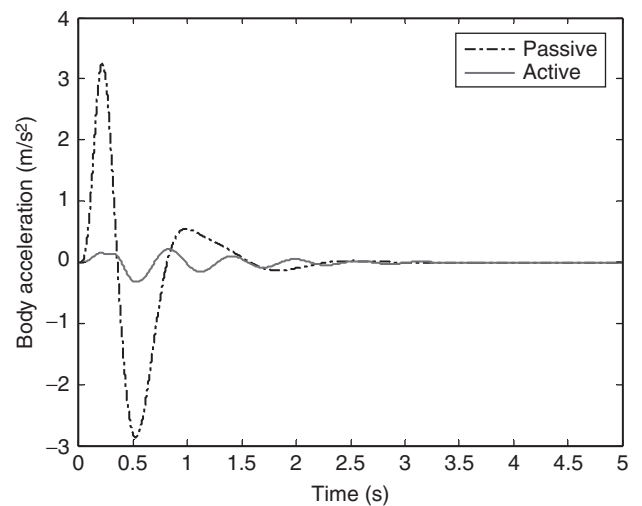
The comparison of the designed static output feedback controllers with passive seat suspension on



**Figure 3.** Bump responses on driver body acceleration for Case 1



**Figure 4.** Bump responses on driver body acceleration for Case 2



**Figure 5.** Bump responses on driver body acceleration for Case 3

bump responses is shown in Figures 3–7, respectively, for five cases. It can be seen that both Case 1, Case 2 and Case 5 do not achieve too much improved ride comfort

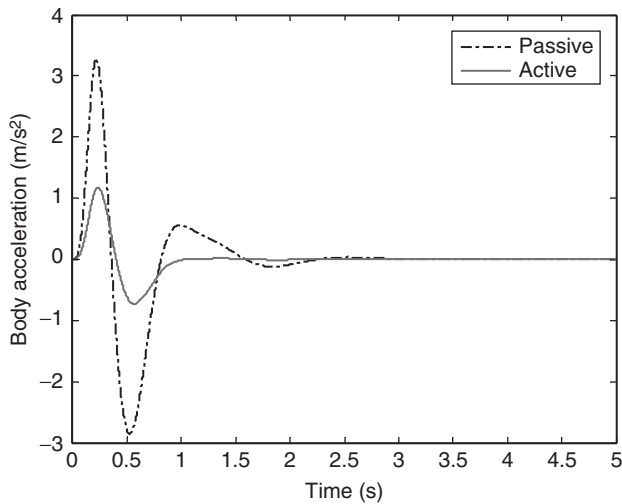


Figure 6. Bump responses on driver body acceleration for Case 4

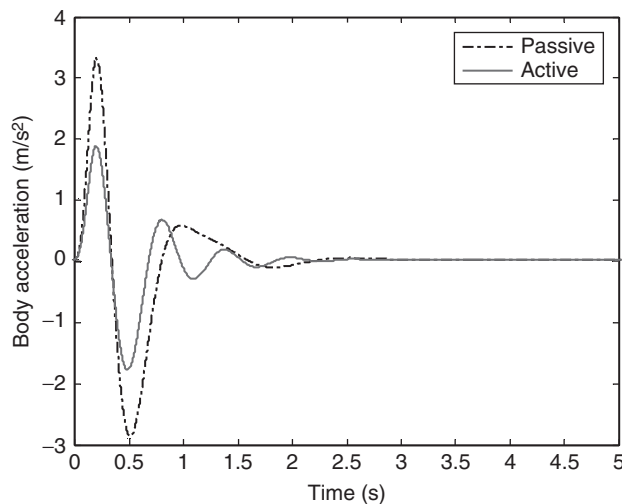


Figure 7. Bump responses on driver body acceleration for Case 5

performance compared the passive seat suspension, however, Case 3 and Case 4 achieve an improved ride comfort performance compared to the passive seat suspension. These results may indicate that we could to choose the most appropriate measurement available signals to provide the best control action so that the ride comfort performance can be improved. This, however, may need further investigation.

At last, we check the robustness of the designed controller by changing the driver body mass. The comparison result between the state feedback controller and the static output feedback controller, which uses car suspension velocity and seat suspension velocity as feedback signals, is shown in Figure 8. The driver body mass is varied from 40 kg to 100 kg, and the ratio is defined as the peak acceleration ratio between the seat suspension with the indicated controller and the passive seat suspension. It can be seen that the static feedback controller can keep the ride comfort performance

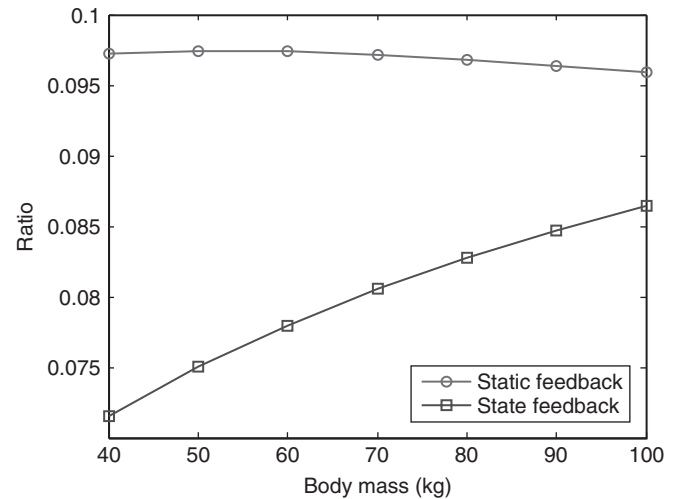


Figure 8. Bump responses on driver body acceleration with varying driver body mass

unchanged in spite of the changes of driver body mass. The ride comfort performance realized by the state feedback controller, which does not consider the driver body mass variation in the controller design process, changes with the variation of driver body mass. The proposed controller shows its better robustness on driver body mass.

To further validate the effectiveness of the proposed control strategy, the system responses under random road disturbance are compared. Samples of the random road profile are generated using the spectral representation method (Verros *et al.* 2005). For brevity, only the comparison on acceleration response under random road with roughness of E Grade (Very Poor) according to ISO 2631 standards and vehicle speed 60 km/h is shown in Figure 9, where static output feedback controller uses seat suspension displacement and velocity as feedback signals. It can be seen from Figure 9 that better

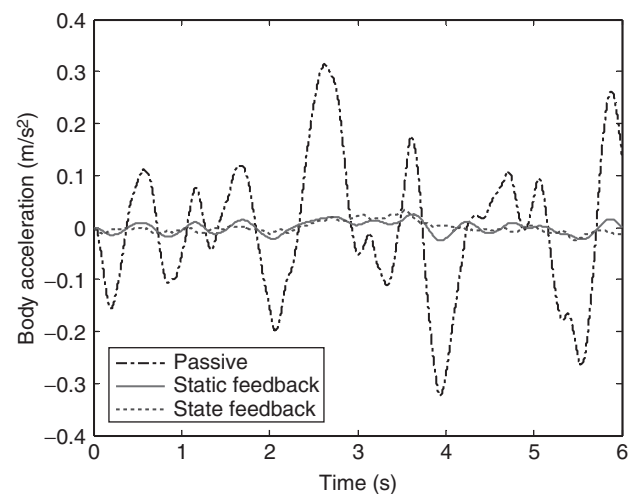


Figure 9. Random responses on driver body acceleration for different control systems

performance is still achieved by the designed static output feedback controller.

## 5. CONCLUSIONS

In this paper, an integrated seat suspension model has been developed and used for active seat suspension design. Static output feedback controller design method is presented with considering driver body mass variation and the limited capability of actuators. Numerical simulations are used to validate the effectiveness of the designed controllers. At first the performance of the designed active seat suspension using a static output feedback controller and a state feedback controller, respectively, is compared with the passive seat suspension under a bump road disturbance. The results show that an appropriately designed static output controller can provide better ride comfort performance compared to the passive seat suspension, and the performance of the static output control is compatible to the state feedback control with a realisable structure. Then different forms of the static output feedback controllers in terms of the measurement available signals are designed and compared to the passive seat suspension. The results show that choosing the appropriate measurement available signals will affect the improvement on the ride comfort performance. At last the robustness of the designed controller is checked by varying the driver body mass, and the validation of the controller's performance over random road disturbances is conducted. All the numerical results show that the designed static output controller is effective on improving the ride comfort performance of the integrated seat suspension. Further study on the optimal configuration of the static output feedback controller considering parameter uncertainties will be investigated.

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## NOTATION

The notation used throughout the paper is fairly standard. For a real symmetric matrix  $W$ , the notation of  $W > 0$  ( $W < 0$ ) is used to denote its positive-(negative-) definiteness.  $\|\cdot\|$  refers to either the Euclidean vector norm or the induced matrix 2-norm.  $I$  is used to denote the identity matrix of appropriate dimensions. To simplify notation,  $*$  is used to represent a block matrix which is readily inferred by symmetry.



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